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Exploring light-cone sum rules for pion and kaon form factors

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Abstract. We analyze the higher-twist effects and the $SU(3)$-flavor symmetry breaking in the correlation functions used to calculate form factors of pseudoscalar mesons in the QCD light-cone sum rule approach. It is shown that the Ward identities for these correlation functions yield relations between twist-4 two- and three-particle distribution amplitudes. In addition to the relations already obtained from the QCD equations of motions, we have found a new one. With the help of these relations, the twist-4 contribution to the light-cone sum rule for the pion electromagnetic form factor is reduced to a very simple form. Simultaneously, we correct a sign error in an earlier calculation. The updated light-cone sum rule prediction for the pion form factor at intermediate momentum transfers is compared with the recent Jefferson Lab data. Furthermore, from the correlation functions with strange-quark currents the kaon electromagnetic form factor and the $K \to \pi$ weak transition form factors are predicted with $O(m_s) \sim O(m_K^2)$ accuracy.

1 Introduction

Accurate knowledge of the pion and kaon light-cone distribution amplitudes (DAs) introduced in the studies of hard exclusive processes in QCD [1] is important for the various frameworks where these DAs are being used. Among the most topical applications one could mention the calculations of exclusive semileptonic and hadronic B-meson transitions into pions and kaons using pQCD [2], QCD factorization [3] or light-cone sum rules [4]. Although there are definite indications that at the normalization scale of $O(1\,\mathrm{GeV})$ the leading twist-2 pion DA is already quite close to its asymptotic shape, one still encounters a large uncertainty of the non-asymptotic part. Moreover, very little is known about non-asymptotic $SU(3)$-flavor asymmetry in the twist-2 kaon DA.

One of the promising ways to study DAs is to employ vacuum-to-pion or vacuum-to-kaon correlation functions of light-quark currents. At high virtualities, using the operator-product expansion (OPE) near the light-cone, these correlation functions are expressed in terms of DAs. On the other hand, the same correlation functions are related, via dispersion relations, to the observable form factors of pions and kaons with the contributions of excited hadronic states approximated by quark–hadron duality. In the resulting relations, known as *light-cone sum rules* (LCSRs) [5], the experimental data on form factors can be used to yield non-trivial constraints on the DAs. The LCSR for the pion electromagnetic (e.m.) form factor was derived in [6,7] and for the $\gamma^*\gamma\pi^0$ form factor in [8]. In order to further increase the accuracy of these sum rules one has to gain a better control over higher-twist effects in the OPE. In the case of the pion form factors the most important subleading contribution to the LCSR is of twist 4. The kaon e.m. form factor, which so far was not analyzed in the LCSR framework, demands also inclusion of the twist-3 effects proportional to m_s.

The aim of this paper is twofold. First, we analyze the higher-twist effects in the vacuum-to-pion and vacuum-to-kaon correlation functions. We demonstrate that a new useful tool is provided by standard Ward identities for the conserved e.m. and axial (in the chiral limit) currents. Simultaneously, we correct a sign error in the previous calculation of the twist-4 term and update the LCSR prediction for the pion e.m. form factor. Second, we include $SU(3)$-flavor symmetry breaking effects at $O(m_s) \sim O(m_K^2)$ in the correlation functions. We calculate the twist-3 part and obtain LCSR for the kaon e.m. and $K \to \pi$ weak transition form factors at intermediate spacelike momentum transfers.

The plan of this paper is as follows. In Sect. 2 we introduce a generic correlation function, which yields LCSRs for the pion, kaon, and $K \to \pi$ form factors for different flavor combinations of light-quark currents. The correlation function is then calculated with twist-4 accuracy including terms of first order in the quark mass. In Sect. 3 we derive the Ward identities in the chiral limit and demonstrate that they lead to relations between two- and three-particle DAs of twist 4. In Sect. 4 the numerical results for the pion form factor are presented with a corrected twist-4 contribution. A comparison of our prediction is made with the recent data on the pion e.m. form factor

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obtained at CEBAF. Section 5 contains LCSR results for the kaon electromagnetic form factor, and Sect. 6 deals with the $K \to \pi$ weak transition form factor. We summarize our conclusions in Sect. 7. The appendices contain the expansion of the quark propagator in Appendix A, the definitions of the DAs and their asymptotic expansions in Appendix B and in Appendix C the α_s corrections to the twist-2 LCSR obtained in [7].

2 Correlation functions

As a starting point, we introduce a generic correlation function:

$$T_{\mu\nu}(p,q) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx}\langle 0|T\Big\{ (\bar{q}_2(0)\gamma_\mu\gamma_5 q_1(0)) \times (e_1\bar{q}_1(x)\gamma_\nu q_1'(x) + e_2\bar{q}_2'(x)\gamma_\nu q_2(x)\Big\}|P(p)\rangle, \quad (1)$$

where, in order to obtain the LCSR for the pion e.m. form factor the following quark-flavor combination has to be taken: $q_1 = q_1' = u$, $q_2 = q_2' = d$. In this case the on-shell hadronic state $P = \pi^+$, and $e_1 = e_u = 2/3$, $e_2 = e_d = -1/3$ are the quark e.m. charges in units of e. To calculate the kaon e.m. form factor, one simply has to replace $d \to s$ and $\pi^+ \to K^+$ above. There are two other physically interesting correlation functions yielding two independent LCSRs for the $K \to \pi$ weak transition form factors obtained from (1) at $q_1 = s$, $q_1' = u$, $q_2 = d$, $P = \pi^+$, $e_1 = 1$, $e_2 = 0$ and at $q_1 = d$, $q_2 = u$, $q_2' = s$, $e_1 = 0$, $e_2 = 1$, $P = K^0$. Summarizing, if one calculates the correlation function (1) the result can easily be adjusted to any of the flavor combinations listed above. If the external four-momenta squared q^2 and $(p-q)^2$ are spacelike and large, the operator product in the correlation function (1) can be expanded near the light-cone in terms of pion or kaon DAs of increasing twists. One may then retain a few first terms in this expansion, keeping in mind that higher twists are suppressed by inverse powers of $Q^2 = -q^2$ and/or $|(p-q)^2|$ (for a more detailed discussion see e.g. [7,9]). There are two leading-order diagrams obtained from the two terms in (1) by contracting the quark fields q_1 with $\bar{q}_1$ and q_2 with $\bar{q}_2$, respectively, and replacing them by the free-quark propagators. The first diagram proportional to e_1 is depicted in Fig. 1a. The second diagram, proportional to e_2, is obtained from the first one by changing the direction of the quark line and replacing the quark-flavor indices $1 \leftrightarrow 2$. The next-to-leading approximation for the quark propagator generates the diagram in Fig. 1b (and its $\sim e_2$ counterpart) which brings three-particle quark–antiquark–gluon DAs of twist 3 and 4 into the game. This diagram is calculated using term of the first order in the gluon field in the light-cone expansion of the quark propagator given in Appendix A. We systematically retain all terms of $O(m_q) \sim O(m_P^2)$ in order to be able to account for $SU(3)$ breaking effects in the LCSRs for the kaon form factors. At the same time, the $O(m_q^2)$ contributions, arising e.g. from the denominators of quark propagators, are neglected.

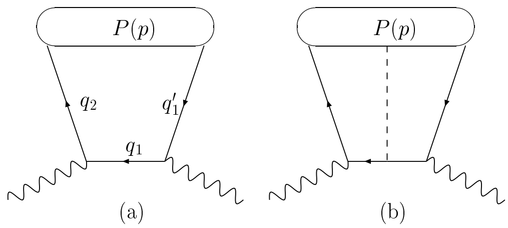

Fig. 1a,b. Diagrams corresponding **a** to the leading order of the correlation function (1); **b** to the contributions of twist-3, -4 quark–antiquark–gluon DAs. Solid, dashed, wavy lines and ovals represent quarks, gluons, external currents and pseudoscalar meson DA, respectively

The result for the correlation function (1) obtained to twist-4 accuracy reads

$$T_{\mu\nu}(p,q) = \mathrm{i}f_P\int_0^1 \mathrm{d}u\Big\{T_1(Q^2,s,u)p_\mu p_\nu + T_2(Q^2,s,u)p_\mu q_\nu + T_3(Q^2,s,u)q_\mu p_\nu + T_4(Q^2,s,u)q_\mu q_\nu + T_5(Q^2,s,u)g_{\mu\nu}\Big\}, \quad (2)$$

with

$$\begin{aligned} T_1(Q^2,s,u) &= \frac{2u\,[e_1\varphi_P(u) - e_2\varphi_P(\bar{u})]}{\bar{u}Q^2 - us + \bar{u}um_P^2} \\ &+ \frac{1}{(\bar{u}Q^2 - us + \bar{u}um_P^2)^2} \\ &\times \Bigg\{ \frac{4f_{3P}}{f_P}\int^u \mathcal{D}\alpha_i\,[e_1 m_{q_1}\varphi_{3P}(\alpha_i) - e_2 m_{q_2}\varphi_{3P}(\bar{\alpha}_i)] \\ &- 2u\Bigg(4\,[e_1 g_{1P}(u) - e_2 g_{1P}(\bar{u})] \\ &- 4\,[e_1 G_{2P}(u) - e_2 G_{2P}(\bar{u})] \\ &- 2u\,[e_1 g_{2P}(u) + e_2 g_{2P}(\bar{u})] \\ &+ \int^u \mathcal{D}\alpha_i\Big[(1-2v)\Big(2[e_1\varphi_{\perp P}(\alpha_i) + e_2\varphi_{\perp P}(\bar{\alpha}_i)] \\ &+ [e_1\varphi_{\parallel P}(\alpha_i) + e_2\varphi_{\parallel P}(\bar{\alpha}_i)]\Big) \\ &- 2[e_1\widetilde{\varphi}_{\perp P}(\alpha_i) - e_2\widetilde{\varphi}_{\perp P}(\bar{\alpha}_i)] \\ &- [e_1\widetilde{\varphi}_{\parallel P}(\alpha_i) - e_2\widetilde{\varphi}_{\parallel P}(\bar{\alpha}_i)]\Big]\Bigg)\Bigg\}, \end{aligned} \quad (3)$$

$$\begin{aligned} T_2(Q^2,s,u) &= -\frac{e_1\varphi_P(u) - e_2\varphi_P(\bar{u})}{\bar{u}Q^2 - us + \bar{u}um_P^2} \\ &+ \frac{1}{(\bar{u}Q^2 - us + \bar{u}um_P^2)^2} \\ &\times \Bigg\{ \frac{\mu_P}{3}\,[e_1 m_{q_1}\varphi_{\sigma P}(u) - e_2 m_{q_2}\varphi_{\sigma P}(\bar{u})] \\ &+ 4\,[e_1 g_{1P}(u) - e_2 g_{1P}(\bar{u})] \\ &- 4\,[e_1 G_{2P}(u) - e_2 G_{2P}(\bar{u})] \\ &- 4u\,[e_1 g_{2P}(u) + e_2 g_{2P}(\bar{u})] \end{aligned}$$

$$
\begin{aligned}
&+ \int^{u} \mathcal{D}\alpha_i \Big[4(1-v)\left[e_1\varphi_{\perp P}(\alpha_i) + e_2\varphi_{\perp P}(\bar\alpha_i)\right] \\
&+ (1-2v)\left[e_1\varphi_{\parallel P}(\alpha_i) + e_2\varphi_{\parallel P}(\bar\alpha_i)\right] \\
&- 4(1-v)\left[e_1\widetilde\varphi_{\perp P}(\alpha_i) - e_2\widetilde\varphi_{\perp P}(\bar\alpha_i)\right] \\
&- \left[e_1\widetilde\varphi_{\parallel P}(\alpha_i) - e_2\widetilde\varphi_{\parallel P}(\bar\alpha_i)\right] \Big] \Big\}, \qquad (4)
\end{aligned}
$$

$$
\begin{aligned}
&T_3(Q^2,s,u) = -\frac{e_1\varphi_P(u) - e_2\varphi_P(\bar u)}{\bar u Q^2 - us + \bar u u m_P^2} \\
&+ \frac{1}{(\bar u Q^2 - us + \bar u u m_P^2)^2} \\
&\times \Big\{ \frac{-\mu_P}{3}\left[e_1 m_{q_1}\varphi_{\sigma P}(u) - e_2 m_{q_2}\varphi_{\sigma P}(\bar u)\right] \\
&+ 4\left[e_1 g_{1P}(u) - e_2 g_{1P}(\bar u)\right] \\
&- 4\left[e_1 G_{2P}(u) - e_2 G_{2P}(\bar u)\right] \\
&- 4u\left[e_1 g_{2P}(u) + e_2 g_{2P}(\bar u)\right] \\
&+ \int^{u} \mathcal{D}\alpha_i \Big[-4v\left[e_1\varphi_{\perp P}(\alpha_i) + e_2\varphi_{\perp P}(\bar\alpha_i)\right] \\
&+ (1-2v)\left[e_1\varphi_{\parallel P}(\alpha_i) + e_2\varphi_{\parallel P}(\bar\alpha_i)\right] \\
&- 4v\left[e_1\widetilde\varphi_{\perp P}(\alpha_i) - e_2\widetilde\varphi_{\perp P}(\bar\alpha_i)\right] \\
&- \left[e_1\widetilde\varphi_{\parallel P}(\alpha_i) - e_2\widetilde\varphi_{\parallel P}(\bar\alpha_i)\right] \Big] \Big\}, \qquad (5)
\end{aligned}
$$

$$
T_4(Q^2,s,u) = 4\frac{\left[e_1 g_{2P}(u) + e_2 g_{2P}(\bar u)\right]}{(\bar u Q^2 - us + \bar u u m_P^2)^2}, \qquad (6)
$$

$$
\begin{aligned}
&T_5(Q^2,s,u) \\
&= -\frac{Q^2 + s + (u-\bar u)m_P^2}{2(\bar u Q^2 - us + \bar u u m_P^2)}\left[e_1\varphi_P(u) - e_2\varphi_P(\bar u)\right] \\
&+ \frac{\mu_P}{(\bar u Q^2 - us + \bar u u m_P^2)}\left[e_1 m_{q_1}\varphi_{pP}(u) - e_2 m_{q_2}\varphi_{pP}(\bar u)\right] \\
&+ \frac{Q^2 + s + (u-\bar u)m_P^2}{2(\bar u Q^2 - us + \bar u u m_P^2)^2} \\
&\times \Big\{ 4\left[e_1 g_{1P}(u) - e_2 g_{1P}(\bar u)\right] \\
&- 4\left[e_1 G_{2P}(u) - e_2 G_{2P}(\bar u)\right] \\
&+ \int^{u} \mathcal{D}\alpha_i \Big[(1-2v)\left[e_1\varphi_{\parallel P}(\alpha_i) + e_2\varphi_{\parallel P}(\bar\alpha_i)\right] \\
&- \left[e_1\widetilde\varphi_{\parallel P}(\alpha_i) - e_2\widetilde\varphi_{\parallel P}(\bar\alpha_i)\right] \Big] \Big\}, \qquad (7)
\end{aligned}
$$

where $s = (p-q)^2$; $\bar u = 1-u$; $\alpha_i = \alpha_1, \alpha_2, 1-\alpha_1-\alpha_2$; $\bar\alpha_i = \alpha_2, \alpha_1, 1-\alpha_1-\alpha_2$ and

$$
\int^{u} \mathcal{D}\alpha_i \equiv \int_0^{u} \mathrm{d}\alpha_1 \int_0^{1-u} \frac{\mathrm{d}\alpha_2}{1-\alpha_1-\alpha_2}, \quad v = \frac{u-\alpha_1}{1-\alpha_1-\alpha_2}.
$$

In the above, φ_P is a generic notation for the twist-2 DA of a pseudoscalar meson $P = \pi$ or K, whereas φ_{pP}, $\varphi_{\sigma P}$, φ_{3P} and g_{1P}, g_{2P}, $\varphi_{\perp P,\parallel P}$, $\tilde\varphi_{\perp P,\parallel P}$ are, respectively, DAs of twist 3 and 4. Their definitions, taken from [10] (see also [11]), are collected in Appendix B. The decay constant f_P of P is defined as $\langle 0 \mid \bar q_2\gamma_\mu\gamma_5 q_1 \mid P(p)\rangle = \mathrm{i}f_P p_\mu$. Furthermore, $\mu_P = m_P^2/(m_{q_1}+m_{q_2})$ is the twist-3 DA normalization factor and $G_{2P}(u) = \int_0^u \mathrm{d}v g_{2P}(v)$. In the case of non-strange quarks, $q_1 = q_1' = u$, $q_2 = q_2' = d$, both chiral and isospin symmetry limits can safely be adopted. In this limit the u, d quark masses as well as the pion mass are neglected and the DAs are either symmetric or antisymmetric (see Appendix B)[1], with respect to the replacements $u \leftrightarrow \bar u$, or $\alpha_1 \leftrightarrow \alpha_2$. In this case the twist-3 parts in (3)–(7) vanish and the combination of quark charges $e_1 - e_2 = e_u - e_d = 1$ factorizes out. The resulting expression for T_1 coincides with the one obtained in [6] except that the signs of the terms containing the twist-4 quark–gluon DA $\varphi_{\perp P,\parallel P}$ are opposite. The same discrepancy in signs is found in the expressions for T_i obtained in the chiral limit in [12] comparing them with (3)–(7). In the next section we will demonstrate that (3)–(7) are fully consistent with the relations obtained from the QCD equations of motion. Finally, we note that the twist-3 terms in (3)–(7) are new.

3 Ward identities

Multiplying the correlation function (1) by the four-momentum q one obtains

$$
\begin{aligned}
q^\nu T_{\mu\nu} = &-\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}qx} \\
&\times \Big(\langle 0|T\Big\{ \bar q_2(0)\gamma_\mu\gamma_5 q_1(0) \\
&\times \frac{\partial}{\partial x_\nu}(e_1\bar q_1(x)\gamma_\nu q_1'(x) + e_2\bar q_2'(x)\gamma_\nu q_2(x)) \Big\}|P(p)\rangle \\
&- \delta(x_0)\langle 0|[\bar q_2(0)\gamma_\mu\gamma_5 q_1(0), \qquad (8) \\
&(e_1\bar q_1(0,\vec x)\gamma_0 q_1'(0,\vec x) + e_2\bar q_2'(0,\vec x)\gamma_0 q_2(0,\vec x))]|P(p)\rangle \Big),
\end{aligned}
$$

where the second term containing equal-time current commutators originates from the differentiation of the $\theta(x_0)$ in the T-product of the currents. For the conserved vector currents $q_1 = q_1'$ and $q_2 = q_2'$ the first term on the r.h.s. of (1) vanishes. For the second term the standard commutation relations for the equal-time current densities can be employed, e.g., in the case of the pion:

$$
\begin{aligned}
&[\bar d(0)\gamma_\mu\gamma_5 u(0), (e_u\bar u(0,\vec x)\gamma_0 u(0,\vec x) + e_d\bar d(0,\vec x)\gamma_0 d(0,\vec x))] \\
&= \delta^{(3)}(\vec x)\bar d(0,\vec x)\gamma_\mu\gamma_5 u(0,\vec x), \qquad (9)
\end{aligned}
$$

yielding for the correlation function the Ward identity

$$
q^\nu T_{\mu\nu} = \mathrm{i}f_\pi p_\mu. \qquad (10)
$$

[1] The type of symmetry is established applying a G-parity transformation to the underlying matrix elements

In the chiral limit $m_{q_1} = m_{q_2} = 0$ the axial-vector current is also conserved. Hence, we get an additional relation:

$$(p-q)^\mu T_{\mu\nu} = -\mathrm{i} f_\pi p_\nu . \qquad (11)$$

The above Ward identities are valid for arbitrary q and p. This circumstance allows one to get relations between various pion DAs by substituting (2) in the l.h.s. of (10) and (11).

Here we will only concentrate on the chiral limit, so that both (10) and (11) are valid and $p^2 = 0$. It is easy to check that the r.h.s. of these equations are saturated by the twist-2 contribution to their l.h.s. Hence, the Ward identities (10) and (11) yield non-trivial relations between two- and three-particle DAs of twist 4. Note that in the chiral limit different twists are separated by dimensions, therefore contributions to the correlation function with twist higher than 4 neglected in our calculation are unimportant[2]. Using

$$\frac{2q.p}{(q-up)^4} = \frac{\partial}{\partial u}\frac{1}{(q-up)^2}$$

and

$$\frac{q^2}{(q-up)^4} = \frac{1}{(q-up)^2} + u\frac{\partial}{\partial u}\frac{1}{(q-up)^2}, \qquad (12)$$

together with partial integration in u, rewriting all twist-4 contributions in (10) and (11) as $\int_0^1 \mathrm{d}u(1/(p-uq)^2)F(u)$ and then extracting $F(u)=0$, one obtains the following relations:

$$g_{2\pi}(u) = \int^u \mathcal{D}\alpha_i \left(\varphi_{\perp\pi}(\alpha_i) - (1-2v)\widetilde{\varphi}_{\perp\pi}(\alpha_i)\right), \qquad (13)$$

$$G_{2\pi}(u) = \frac{u}{2}g_{2\pi}(u) - \frac{1}{2}\int^u \mathcal{D}\alpha_i v\left(\varphi_{\perp\pi}(\alpha_i) + \widetilde{\varphi}_{\perp\pi}(\alpha_i)\right), \qquad (14)$$

$$\begin{aligned} g_{1\pi}(u) = {} & G_{2\pi}(u) + \frac{1}{2}u\bar{u}g'_{2\pi}(u) \\ & - \frac{1}{4}\int^u \mathcal{D}\alpha_i\Big[(1-2v)\left(\varphi_{\parallel\pi}(\alpha_i) + 2\varphi_{\perp\pi}(\alpha_i)\right) \\ & - \widetilde{\varphi}_{\parallel\pi}(\alpha_i) - 2\widetilde{\varphi}_{\perp\pi}(\alpha_i)\Big], \end{aligned} \qquad (15)$$

where $g'_{2\pi}(u) = \partial g_2(u)/\partial u$. We notice that the above expressions can be used to rewrite in the chiral limit ($P=\pi$) the twist-4 part of the correlation function (2) using only one DA $g_{2\pi}$ and its derivative over u, so that

$$\begin{aligned} T_{\mu\nu} = {} & \mathrm{i}f_\pi \int_0^1 \mathrm{d}u \bigg\{ \frac{1}{\bar{u}Q^2 - us} \\ & \times \left(2up_\mu p_\nu - q_\mu p_\nu - p_\mu q_\nu - (q\cdot p)g_{\mu\nu}\right)\varphi_\pi(u) \\ & + \frac{2}{(\bar{u}Q^2-us)^2}\Big\{ p_\mu p_\nu (2u^2 g_{2\pi}(u) - 2u^2\bar{u}g'_{2\pi}(u)) \\ & + (p_\mu q_\nu + q_\mu p_\nu)(-2ug_{2\pi}(u) + u\bar{u}g'_{2\pi}(u)) \\ & + (p_\mu q_\nu - q_\mu p_\nu + 2q_\mu q_\nu)g_{2\pi}(u) \\ & + g_{\mu\nu}\Big[\left(2Q^2 + (q.p)(1+2u)\right)g_{2\pi}(u) \\ & - (q.p)u\bar{u}g'_{2\pi}(u)\Big]\Big\}\bigg\}. \end{aligned} \qquad (16)$$

[2] In fact, we also neglect four-particle Fock components of twist 4 in the light-cone expansion of the matrix elements. This is consistent with the approximation adopted in deriving the relations from the QCD equations of motion [10]

The relations (13) and (14) can also be obtained using the technique of the QCD equations of motion [10]. The starting objects in this case are the derivatives of quark–antiquark operators expressed via quark–antiquark–gluon operators, e.g.:

$$\begin{aligned} & \frac{\partial}{\partial x_\mu}\left\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(x)|\pi^+(p)\right\rangle \\ & = \mathrm{i}\int_0^1 \alpha\mathrm{d}\alpha \left\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 x_\lambda G^{\lambda\mu}(\alpha x)u(x)|\pi^+(p)\right\rangle \end{aligned} \qquad (17)$$

and

$$\begin{aligned} & \frac{\partial}{\partial x_\nu}\left\langle 0|\bar{d}(0)\gamma_\mu\gamma_\beta\gamma_\nu\gamma_5 u(x)|\pi^+(p)\right\rangle \qquad (18) \\ & = \mathrm{i}\int_0^1 \alpha\mathrm{d}\alpha \left\langle 0|\bar{d}(0)\gamma_\mu\gamma_\beta\gamma_\nu\gamma_5 x_\lambda G^{\lambda\nu}(\alpha x)u(x)|\pi^+(p)\right\rangle, \end{aligned}$$

where $G_{\mu\nu} = g_\mathrm{s} G^a_{\mu\nu}(\lambda_a/2)$, $\mathrm{tr}\lambda_a\lambda_b = 2\delta^{ab}$. The relations derived from (17) are

$$\begin{aligned} g_{1\pi}(u) = {} & \frac{u}{2}g_{2\pi}(u) - G_{2\pi}(u) \\ & + \frac{1}{2}\int^u \mathcal{D}\alpha_i v\left(\varphi_{\parallel\pi}(\alpha_i) - 2\varphi_{\perp\pi}(\alpha_i)\right), \end{aligned} \qquad (19)$$

and its $u \leftrightarrow \bar{u}$ equivalent:

$$\begin{aligned} g_{1\pi}(u) = {} & -\frac{\bar{u}}{2}g_{2\pi}(u) - G_{2\pi}(u) \\ & - \frac{1}{2}\int^u \mathcal{D}\alpha_i (1-v)\left(\varphi_{\parallel\pi}(\alpha_i) - 2\varphi_{\perp\pi}(\alpha_i)\right). \end{aligned} \qquad (20)$$

Combining (19) and (20) one gets the two relations obtained in [10]. Equation (18), which was also used in [12], yields

$$\begin{aligned} g_{1\pi}(u) = {} & -\frac{u}{2}g_{2\pi}(u) + G_{2\pi}(u) \\ & + \frac{1}{2}\int^u \mathcal{D}\alpha_i v\left(\varphi_{\parallel\pi}(\alpha_i) + 2\widetilde{\varphi}_{\perp\pi}(\alpha_i)\right), \end{aligned} \qquad (21)$$

and

$$\begin{aligned} g_{1\pi}(u) = {} & \frac{\bar{u}}{2}g_{2\pi}(u) + G_{2\pi}(u) \qquad (22) \\ & + \frac{1}{2}\int^u \mathcal{D}\alpha_i (1-v)\left(-\varphi_{\parallel\pi}(\alpha_i) + 2\widetilde{\varphi}_{\perp\pi}(\alpha_i)\right). \end{aligned}$$

Combining (21) and (22) with (19) and (20), after simple algebra one indeed reproduces the relations obtained from the Ward identities, but only two of them, (13) and (14). The relation (15), the only one involving the DA $\widetilde{\varphi}_{\parallel\pi}$ is new and was not obtained using the equations of motion. Note that the observed consistence between the relations derived from Ward identities and from QCD equations of motion provides an independent check of our result for the correlation function. Indeed, taking the correlation function calculated in [12] with different signs at the terms with $\varphi_{\perp\pi,\parallel\pi}$ we obtain a contradiction between the two types of relations.

If the chiral symmetry is violated, $m_q \sim m_P^2 \neq 0$, the Ward identity (10) based on the conservation of the e.m. current is still valid, but the relations following from this identity are more complicated, mixing DAs of twist 2, 3 and 4 and not allowing one to reduce the twist-4 part to an integral over a single DA. The corresponding analysis goes beyond the scope of this paper.

4 Updated prediction for the pion e.m. form factor

The LCSR for the pion e.m. form factor was originally derived in [6]. Let us briefly outline the procedure. The part of the correlation function (2) (in the chiral limit) proportional to $\sim p_\mu p_\nu$ was matched to the hadronic dispersion relation in the variable $s = (p-q)^2$; that is, in the channel of the axial-vector current:

$$\mathrm{i}f_\pi \int\limits_0^1 \mathrm{d}u T_1(Q^2, s, u) = \frac{2\mathrm{i}f_\pi F_\pi(Q^2)}{-s} + \int\limits_{s_0^h}^{\infty} \frac{\rho^h(s')\mathrm{d}s'}{s'-s}. \quad (23)$$

In this relation, the first term on the r.h.s. is the ground-state contribution of the pion where $f_\pi = 132\,\mathrm{MeV}$ and $F_\pi(Q^2)$ is the pion e.m. form factor defined in the standard way:

$$F_\pi(Q^2)(2p-q)_\nu = \langle \pi^+(p-q) \mid j_\nu^{\mathrm{em}} \mid \pi^+(p)\rangle, \quad (24)$$

j_ν^{em} being the quark e.m. current. The contributions of the a_1 meson and excited states with $J^P = 0^-, 1^+$ form the spectral density ρ^h which is estimated as usual, with the help of quark–hadron duality:

$$\rho^h(s)\Theta(s-s_0^h) = \frac{\mathrm{i}f_\pi}{\pi} \int\limits_0^1 \mathrm{d}u \mathrm{Im}_s T_1(Q^2, s, u)\Theta(s-s_0^\pi), \quad (25)$$

where the effective threshold parameter $s_0^\pi = 0.7\,\mathrm{GeV}^2$ is determined from the SVZ sum rule [13] for the correlator of two $\bar{u}\gamma_\mu\gamma_5 d$ currents. Representing the l.h.s of (23) in the form of the dispersion integral:

$$\int\limits_0^1 \mathrm{d}u T_1(Q^2, s, u) = \frac{1}{\pi} \int\limits_0^{\infty} \frac{\mathrm{d}s'}{s'-s} \int\limits_0^1 \mathrm{d}u \mathrm{Im}_{s'} T_1(Q^2, s', u), \quad (26)$$

using (25) and performing the Borel transformation, $(p-q)^2 \to M^2$, we obtain the resulting sum rule:

$$F_\pi(Q^2) = \frac{1}{2\pi} \int\limits_0^{s_0^\pi} \mathrm{d}s \mathrm{e}^{-s/M^2} \int\limits_0^1 \mathrm{d}u \mathrm{Im}_s T_1(Q^2, s, u). \quad (27)$$

In the twist-2 approximation one has [6]

$$F_\pi^{(2)}(Q^2) = \int_{u_0^\pi}^1 \mathrm{d}u \varphi_\pi(u, \mu) \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right). \quad (28)$$

In the above $u_0^\pi = Q^2/(s_0^\pi + Q^2)$ and we have indicated the dependence of the DA φ_π on the normalization scale μ.

The LCSR (27) was further improved in [7] where the $O(\alpha_\mathrm{s})$ contribution to the twist-2 part was calculated by taking into account the perturbative gluon exchanges between the quark lines in the diagram of Fig. 1a. For convenience we present in Appendix C the explicit expression for $F_\pi^{(2,\alpha_\mathrm{s})}(Q^2)$ which has to be added to the r.h.s. of (28). Recall that this contribution provides the $\sim \alpha_\mathrm{s}/Q^2$ asymptotic behavior [1] of the form factor. As explained in detail in [7] the form factor obtained from LCSR includes both the hard-scattering and soft (end-point) contributions.

Our main update of the sum rule for $F_\pi(Q^2)$ concerns the twist-4 term for which a new, corrected expression is obtained from (16):

$$F_\pi^{(4)}(Q^2) = \int\limits_{u_0^\pi}^1 \mathrm{d}u \frac{\varphi_\pi^{(4)}(u, \mu)}{uM^2} \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right) + \frac{u_0^\pi \varphi_\pi^{(4)}(u_0^\pi, \mu)}{Q^2} \mathrm{e}^{-s_0^\pi/M^2}, \quad (29)$$

where

$$\varphi_\pi^{(4)}(u, \mu) = 2u\left(g_{2\pi}(u, \mu) - \bar{u}g'_{2\pi}(u, \mu)\right). \quad (30)$$

The second term on the r.h.s. of (29) is a "surface term" originating after the continuum subtraction as explained in [7]. In the same paper the factorizable twist-6 contribution to LCSR was calculated:

$$F_\pi^{(6)}(Q^2) = \frac{4\pi\alpha_\mathrm{s} C_F}{3f_\pi^2 Q^4} \langle 0 \mid \bar{q}q \mid 0\rangle^2 \quad (31)$$

in terms of the quark condensate density (see [7] for the diagrams and other details concerning this contribution). Note that the twist-6 term is numerically very small starting from $Q^2 = 1\,\mathrm{GeV}^2$ which is therefore a natural lower boundary of the LCSR validity region[3].

We turn now to the numerical calculation of the pion form factor,

$$F_\pi(Q^2) = F_\pi^{(2)}(Q^2) + F_\pi^{(2,\alpha_\mathrm{s})}(Q^2) + F_\pi^{(4)}(Q^2) + F_\pi^{(6)}(Q^2), \quad (32)$$

[3] Recent work on the pion and kaon form factors at low momentum transfers can be found in [14]

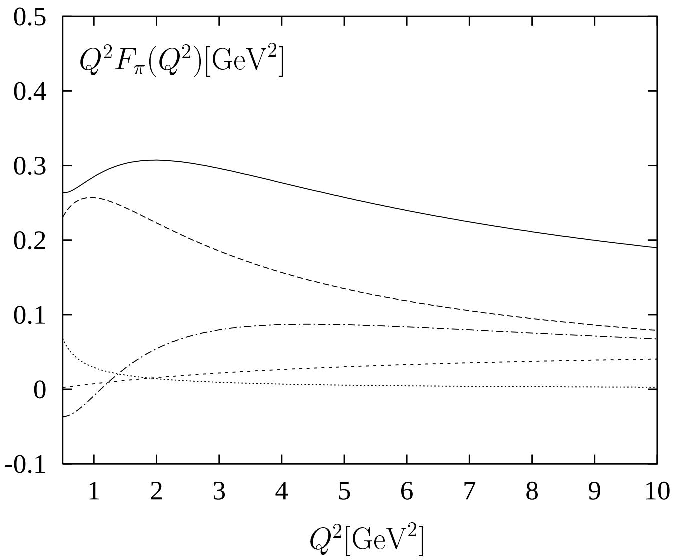

Fig. 2. Pion e.m. form factor obtained from LCSR with the asymptotic pion DA (*solid*) including the twist-2 leading-order (*long-dashed*), twist-2 $O(\alpha_s)$ (*short-dashed*), twist-4 (*dash-dotted*) and factorizable twist-6 (*dotted*) contributions

where twist-2, -4, and the factorizable twist-6 contributions to LCSR are added together. In our numerical evaluation of (32), following [7] we take $0.8 < M^2 < 1.5\,\mathrm{GeV}^2$ and adopt the variable normalization scale $\mu_u^2 = (1-u)Q^2 + uM^2$ of the light-cone DA. The same scale is adopted for the normalization of α_s. For the latter the two-loop running is used with $\bar{\Lambda}^{(3)} = 340\,\mathrm{MeV}$ corresponding to $\alpha_s(1\,\mathrm{GeV}) = 0.48$. For the twist-2 pion DA we take the asymptotic form $\varphi_\pi(u) = 6u(1-u)$. The influence of non-asymptotic corrections will be discussed later. Concerning the twist-4 pion DAs: we actually need only one of them, $g_{2\pi}$. Interestingly, in first order of the conformal expansion [10] this DA does not contain non-asymptotic contributions. Using the asymptotic form of $g_{2\pi}(u)$ presented in Appendix B one obtains a compact expression:

$$\varphi_\pi^{(4)}(u,\mu) = \frac{20}{3}\delta_\pi^2(\mu)u\bar{u}(1-u(7-8u)). \qquad (33)$$

The non-perturbative parameter $\delta_\pi^2 \simeq 0.2\,\mathrm{GeV}^2$ determining the vacuum-to-pion matrix element of the quark–antiquark–gluon current (see the definition in Appendix B) was estimated from various two-point QCD sum rules in [15]. To assess the theoretical uncertainty, we have recalculated δ_π^2 using the diagonal sum rule for two quark–antiquark–gluon currents which is less dependent on the variations of quark and gluon condensates. The result, in agreement with [15], is

$$\delta_\pi^2(1\,\mathrm{GeV}) = 0.17 \pm 0.05\,\mathrm{GeV}^2, \qquad (34)$$

obtained with $\langle 0 \mid \bar{q}q \mid 0\rangle = (-240 \pm 10\,\mathrm{MeV})^3$ and $\langle 0 \mid (\alpha_s/\pi)G^a_{\mu\nu}G^{a\mu\nu} \mid 0\rangle = 0.012 \pm 0.006\,\mathrm{GeV}^4$.

Our prediction for the pion e.m. form factor given by (32) is plotted in Fig. 2, calculated with the asymptotic pion DA at the typical value of $M^2 = 1\,\mathrm{GeV}^2$, and at $\mu = \mu_u$ and $\delta_\pi^2 = 0.17\,\mathrm{GeV}^2$. Importantly, the corrected twist-4 contribution is about two times larger than estimated before [6,7]. Note that at $Q^2 \to \infty$ the twist-4 term given by (29) has the same $\sim 1/Q^4$ asymptotic behavior as the twist-2 contribution (28)[4], but has one extra power of $1/M^2$. This can be seen explicitly by rewriting (29), with the help of (33), in the form of a dispersion integral with the integration variable $s = Q^2\bar{u}/u$:

$$F_\pi^{(4)}(Q^2) = \frac{40}{3}\delta_\pi^2(\mu)\int\limits_0^{s_0^\pi} \mathrm{d}s\mathrm{e}^{-s/M^2}\frac{Q^8}{(Q^2+s)^6} \times\left(1 - \frac{9s}{Q^2} + \frac{9s^2}{Q^4} - \frac{s^3}{Q^6}\right), \qquad (35)$$

yielding at $Q^2 \to \infty$

$$F_\pi^{(4)}(Q^2) \sim \frac{40\delta_\pi^2 M^2}{3Q^4}\left(1 - \mathrm{e}^{-s_0^\pi/M^2}\right), \qquad (36)$$

to be compared with the corresponding limit of (28):

$$F_\pi^{(2)}(Q^2) \sim \frac{6M^4}{Q^4}\left(1 - \left(1 + \frac{s_0^\pi}{M^2}\right)\mathrm{e}^{-s_0^\pi/M^2}\right). \qquad (37)$$

Although the twist-4 term has indeed an extra suppression factor δ_π^2/M^2 as compared with the twist-2 term, the overall numerical coefficients in (36) and (37) are of the same order at $M^2 \sim 1\,\mathrm{GeV}^2$.

The LCSR approach allows one to estimate the theoretical uncertainty of the predicted form factor. We did it in the following way. First of all, M^2 and δ_π^2 were varied within allowed intervals. Furthermore, in order to investigate the sensitivity to the choice of the renormalization scale, our calculation was repeated at two fixed scales Q^2 and M^2 adopting the variation of the results as a theoretical uncertainty. Finally, accounting for the missing twist $\geq$6 terms we assume that the absence of the latter introduces an additional uncertainty equal to $\pm F_\pi^{(6)}(Q^2)$. All abovementioned variations of the LCSR prediction for $F_\pi(Q^2)$ are then added linearly, which is a rather conservative approach.

In Fig. 3 we plot $F_\pi(Q^2)$, calculated with the asymptotic pion DA and at $M^2 = 1\,\mathrm{GeV}^2$. The resulting uncertainty of the form factor indicated in this figure is about $\pm(20 \div 30)\%$ at $Q^2 \geq 1\,\mathrm{GeV}^2$. At $Q^2 < 1\,\mathrm{GeV}^2$ the uncertainty grows revealing the inapplicability of LCSR for small momentum transfers. In the region $2.0 \leq Q^2 < 10\,\mathrm{GeV}^2$ our prediction for the pion e.m. form factor with the asymptotic pion DA can be fitted to the following simple formula:

$$Q^2F_\pi(Q^2) = (0.0735 \div 0.2016) + \frac{0.7908 \div 0.9340}{Q^2} - \frac{0.8496 \div 1.2068}{Q^4} \qquad (38)$$

[4] Contributions non-vanishing or growing with Q^2 are absent in LCSR. Such anomalous contributions emerge in QCD sum rules based on the local condensate expansion [16,17] making the latter not applicable at $Q^2 \gg 1\,\mathrm{GeV}^2$

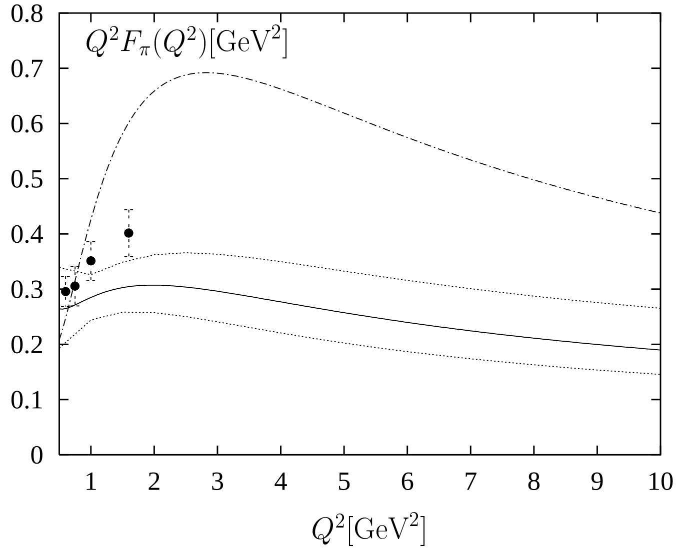

Fig. 3. The pion e.m. form factor calculated from LCSR in comparison with the Jefferson Lab data [18] shown with points (the experimental error and the model uncertainty are added in quadratures). The *solid line* corresponds to the asymptotic pion DA, *dashed lines* indicate the estimated overall theoretical uncertainty; the *dash-dotted line* is calculated with the CZ model [19] of the pion DA

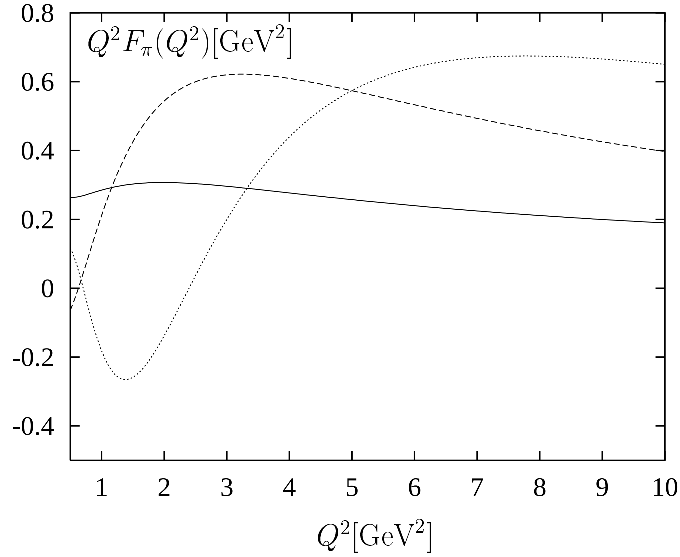

Fig. 4. Graphical illustration of (39). The pion e.m. form factor obtained from LCSR with the asymptotic pion DA (*solid line*) and the coefficients at $a_2(1\,\mathrm{GeV})$ (*long-dashed*) and $a_4(1\,\mathrm{GeV})$ (*short-dashed*)

(all numbers in GeV^2), where the correlated intervals take into account the total uncertainty.

The accuracy of the LCSR prediction (32) can be improved further by including various higher-twist corrections (due to twist-4 multiparticle and twist-6 DAs) which were not yet analyzed. However, the smallness of the factorizable twist-6 term indicates that these effects are most probably numerically unimportant. In addition, it is desirable to improve also the perturbative expansion of the correlation function calculating the $O(\alpha_s)$ term of twist 4 and the $O(\alpha_s^2)$ term of twist 2. An attempt to account for the latter was made in [7] by matching LCSR to the NLO perturbative calculation [20].

Finally, in Fig. 3 we compare our numerical prediction with the recent accurate data on $F_\pi(Q^2)$ obtained from the pion electroproduction at Jefferson Lab [18] at $Q^2 = 0.6 \div 1.65\,\mathrm{GeV}^2$, in the region which only partly overlaps the LCSR validity region $Q^2 > 1\,\mathrm{GeV}^2$. We find that within theoretical uncertainties and experimental errors the form factor calculated with the asymptotic pion DA $\varphi_\pi(u)$ is consistent with data[5].

With $F_\pi(Q^2)$ accurately measured at the whole region $Q^2 = 1 \div 10\,\mathrm{GeV}^2$ it should in principle be possible to constrain/fit the non-asymptotic part of $\varphi_\pi(u)$ determined by the coefficients a_n in the expansion over Gegenbauer polynomials (see Appendix B). Taking into account the complete expansion one obtains

$$F_\pi(Q^2) = [F_\pi(Q^2)]_{as} + \sum_{n=1}^{\infty} a_{2n}(\mu_0) f_{2n}(Q^2,\mu,\mu_0), \quad (39)$$

where the first term on the r.h.s. is the form factor calculated with the asymptotic DA and in the sum each a_{2n} is multiplied by a calculable function:

$$f_{2n}(Q^2,\mu,\mu_0) = 6\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{-\gamma_n^{(0)}/\beta_0} \times \int_{u_0^\pi}^1 du u\bar{u} C_{2n}^{3/2}(u-\bar{u}) \exp\left(-\frac{\bar{u}Q^2}{uM^2}\right) + \cdots \quad (40)$$

In the above, $\mu_0 \sim 1\,\mathrm{GeV}$ is a certain low scale, and the anomalous dimensions γ_n of the renormalization factors are given in Appendix B. For brevity, the $O(\alpha_s)$ correction to (40) is denoted by an ellipsis.

A direct fit of all a_n from (39) is of course not a realistic task. In fact, using the arguments of the conformal partial wave expansion, one expects that the coefficients are decreasing with n, $a_{2n+2} < a_{2n}$. Based on these arguments, the form of $\varphi_\pi(u)$ usually discussed in the literature involves one (a_2) or two (a_2, a_4) non-zero coefficients neglecting the rest. Having adopted a certain simple ansatz for $\varphi_\pi(u)$ one is then able to constrain or even fit the coefficients from (39). However, the current data [18] are sufficient to constrain only the simplest ansatz with a single non-zero coefficient a_2. This can be seen from Fig. 4 where f_2 and f_4 in (40) are plotted in comparison with F_π^{as}. Due to different signs of f_2 and f_4 at $Q^2 \leq 3\,\mathrm{GeV}^2$ it is difficult to distinguish the form of φ_π with $a_2 \neq 0$ from the one where both $a_2, a_4 \neq 0$. For instance, one obtains equally good fits to the experimental data shown in Fig. (3) with $a_2(1\,\mathrm{GeV}) = 0.05$, $a_4(1\,\mathrm{GeV}) = -0.30$ as with $a_2(1\,\mathrm{GeV}) = 0.25, a_4(1\,\mathrm{GeV}) = 0$. If we impose that

[5] LCSR predictions also agree with older measurements of $F_\pi(Q^2)$ at $Q^2 = 1 \div 6\,\mathrm{GeV}^2$, which however have large experimental errors

all $a_{n>2} = 0$ in (39) the coefficient a_2 can be fitted to the following interval consistent with zero:

$$a_2(1\,\mathrm{GeV}) = 0.24 \pm 0.14 \pm 0.08, \tag{41}$$

where the first error reflects our estimated theoretical uncertainty, whereas the second one corresponds to the experimental errors. One needs data at larger Q^2 to resolve more complicated patterns of the non-asymptotic coefficients in φ_π.

5 The kaon electromagnetic form factor

The LCSR for the charged kaon e.m. form factor can be easily obtained from the correlation function (3), substituting $P = K^+$, $e_1 = e_u = +2/3$, $e_2 = e_s = -1/3$, $m_{q_1} = 0$, $m_{q_2} = m_s$. We will systematically retain all $O(m_s) \sim O(m_K^2)$ effects, which are numerous, in general. At the purely kinematical level one has to account for $p^2 = m_K^2$ in the correlation function. Furthermore, the s-quark propagator produces a chirally non-invariant part proportional to m_s which brings the twist-3 contribution into the game (the m_s^2 in the denominator of the quark propagator is neglected, being a higher-order effect). Finally, in the light-cone DA there are $SU(3)$-flavor symmetry ($SU(3)_{\mathrm{fl}}$) violating corrections of three types. First, the normalization factors, determining the quark–antiquark vacuum-to-kaon matrix elements in the local limit $x = 0$ differ from the corresponding factors for the pionic matrix elements, e.g. $f_K \neq f_\pi$. Secondly, the non-asymptotic parts of the kaon DA are asymmetric with respect to the interchange of quark and antiquark fields, with a larger average momentum fraction of the strange quark. At the twist-2 level this effect manifests itself in the non-vanishing odd coefficients of the Gegenbauer expansion (B.7): $a_1^K, a_3^K, \dots \sim m_s \neq 0$. For the higher-twist DAs the $SU(3)_{\mathrm{fl}}$ violating asymmetries were not studied yet, and in our numerical calculation we will neglect them. On the other hand, we will take into account the so-called meson-mass corrections to the twist-4 DAs investigated and worked out in [11]. These effects include the mixing of non-asymptotic parts of twist-2, -3 and -4 DAs beyond the chiral limit. The corresponding expressions are presented in Appendix B.

Apart from $SU(3)_{\mathrm{fl}}$ violating corrections listed above the derivation of the LCSR for the kaon e.m. form factor repeats the procedure for the pion outlined in the previous section. The result reads

$$\begin{aligned} F_K(Q^2) &= F_K^{(2)}(Q^2) + F_K^{(2,\alpha_s)}(Q^2) \\ &\quad + F_K^{(3)}(Q^2) + F_K^{(4)}(Q^2) + F_K^{(6)}(Q^2), \end{aligned} \tag{42}$$

where the twist-2 contribution is

$$\begin{aligned} F_K^{(2)}(Q^2) &= \int\limits_{u_0^K}^{1} \mathrm{d}u \left(\frac{2}{3}\varphi_K(u,\mu) + \frac{1}{3}\varphi_K(\bar u,\mu)\right) \\ &\quad \times \exp\left(-\frac{\bar u Q^2}{uM^2} + \frac{u m_K^2}{M^2}\right). \end{aligned} \tag{43}$$

The DA $\varphi_K(u,\mu)$ is defined as in (B.1), with $q_1 = u$ and $q_2 = s$, so that $\bar u$ is the momentum fraction of the s-quark. In the above, the lower limit u_0^K is related to the duality threshold in the kaon channel s_0^K by the equation $s_0^K = \bar u_0^K (Q^2/u_0^K + m_K^2)$ which should be solved with $O(m_K^2)$ accuracy. The $O(\alpha_s)$ correction to the twist-2 contribution has been calculated in [7] in the chiral limit. To obtain $F_K^{(2,\alpha_s)}$ we replace φ_π by φ_K in the expression for $F_\pi^{(2,\alpha_s)}$ given in Appendix C. In addition, there are $SU(3)_{\mathrm{fl}}$ violating corrections in the hard amplitude. Note that the first-order in m_s corrections are absent due to chirality. Nevertheless, indirectly, $O(m_s)$ contributions will appear due to purely kinematical terms $O(p^2 = m_K^2)$. To obtain these terms one has to recalculate the $O(\alpha_s)$ diagrams retaining $p^2 \neq 0$, which is beyond the task of this paper.

The twist-3 and -4 terms in (42) can be cast in the same form as the twist-4 contribution to the pion form factor:

$$\begin{aligned} F_K^{(3,4)}(Q^2) &= \int\limits_{u_0^K}^{1} \mathrm{d}u \frac{\varphi_K^{(3,4)}(u,\mu)}{uM^2} \exp\left(-\frac{\bar u Q^2}{uM^2} + \frac{u m_K^2}{M^2}\right) \\ &\quad + \left(\frac{1}{Q^2 + s_0^K} + \frac{(Q^2 - s_0^K) m_K^2}{(Q^2 + s_0^K)^3}\right) \\ &\quad \times \varphi_K^{(3,4)}(u_0^K,\mu) \mathrm{e}^{-s_0^K/M^2 + m_K^2/M^2}, \end{aligned} \tag{44}$$

where

$$\varphi_K^{(3)}(u,\mu) = \frac{2 m_s f_{3K}}{3 f_K u} \int\limits^{u} \mathcal{D}\alpha_i \varphi_{3K}(\alpha_i) \tag{45}$$

and

$$\begin{aligned} \varphi_K^{(4)}(u,\mu) = -\frac{1}{3} \Bigg(& 4\left[2 g_{1K}(u) + g_{1K}(\bar u)\right] \\ & - 4\left[2 G_{2K}(u) + G_{2K}(\bar u)\right] \\ & - 2u\left[2 g_{2K}(u) - g_{2K}(\bar u)\right] \\ & + \int\limits^{u} \mathcal{D}\alpha_i \Big[(1-2v)\Big(2\left[2\varphi_{\perp K}(\alpha_i) - \varphi_{\perp K}(\bar\alpha_i)\right] \\ & + \left[2\varphi_{\parallel K}(\alpha_i) - \varphi_{\parallel K}(\bar\alpha_i)\right]\Big) \\ & - 2\left[2\widetilde\varphi_{\perp K}(\alpha_i) + \widetilde\varphi_{\perp K}(\bar\alpha_i)\right] \\ & - \left[2\widetilde\varphi_{\parallel K}(\alpha_i) + \widetilde\varphi_{\parallel K}(\bar\alpha_i)\right]\Big]\Bigg). \end{aligned} \tag{46}$$

In the surface term in (44) only $O(m_K^2)$ terms are taken into account in accordance with our approximation. Since the twist-3 contribution to the correlation function is proportional to m_s it is consistent to use the $SU(3)_{\mathrm{fl}}$ limit of the twist-3 DA, in particular $\varphi_{3K} = \varphi_{3\pi}$ in (45). Finally, for simplicity we adopt the $SU(3)_{\mathrm{fl}}$ limit for the numerically small twist-6 factorizable term.

In Fig. 5 we plot the kaon e.m. form factor calculated with the following choice of parameters:

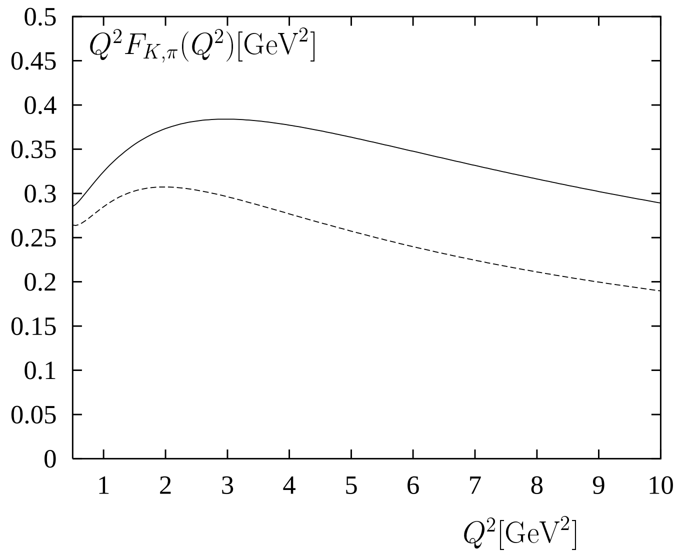

Fig. 5. The LCSR prediction for the charged kaon e.m. form factor $F_K(Q^2)$ (*solid line*) in comparison with the pion form factor obtained with the asymptotic pion DA (*dashed line*) at $M^2 = 1\,\mathrm{GeV}^2$

(a) the twist-2 DA is taken with $a_1^K(1\,\mathrm{GeV}) = -0.17$ (as estimated from the two-point sum rule in [19]) and neglecting all higher Gegenbauer coefficients (in particular $a_2^K = 0$), that is, maximally close to the asymptotic regime;
(b) $s_0^K = 1.2\,\mathrm{GeV}^2$ is determined from the two-point QCD sum rules for f_K [21]. Importantly, s_0^K is larger than s_0^π reflecting heavier states in the kaon channel;
(c) for the strange quark mass we adopt an interval $m_s(1\,\mathrm{GeV}) = 150 \pm 50\,\mathrm{MeV}$;
(d) the parameters of twist-3 and -4 kaon DAs taken in the $SU(3)_{\mathrm{fl}}$ limit[6]: $f_{3K}(1\,\mathrm{GeV}) = f_{3\pi}(1\,\mathrm{GeV}) = 0.0035\,\mathrm{GeV}^2$, $\delta_K^2(1\,\mathrm{GeV}) = \delta_\pi^2(1\,\mathrm{GeV}) = 0.17 \pm 0.05\,\mathrm{GeV}^2$ (see (34)), $\omega_{3K}(1\,\mathrm{GeV}) = \omega_{3\pi}(1\,\mathrm{GeV}) = -2.88$, $\epsilon_{4K}(1\,\mathrm{GeV}) = \epsilon_\pi(1\,\mathrm{GeV}) = 0.5$ [10,11]. The typical accuracy of all parameters except δ_π^2 is about 50%. We also use $f_K = 1.22 f_\pi$.

Comparing the LCSR prediction for the pion and kaon form factors calculated at $M^2 = 1\,\mathrm{GeV}^2$ we observe a noticeable $SU(3)_{\mathrm{fl}}$ violating difference. The ratio $F_K(Q^2)/F_\pi(Q^2)$ approaches 1.5 at $Q^2 \sim 10\,\mathrm{GeV}^2$. A closer look at (43) reveals that this difference originates from an interplay of two opposite effects. The $SU(3)_{\mathrm{fl}}$ asymmetry due to $a_1 \neq 0$ in $\varphi_K(u)$ tends to suppress the kaon form factor because in the larger contribution (corresponding to the u-quark interacting with the virtual photon) $\varphi_K(u) < \varphi_\pi(u)$ in the end-point integration region $u_0^K < u < 1$. On the other hand, the fact that the duality threshold for the kaon is higher, $s_{0K} > s_{0\pi}$, implies that the end-point region for the kaon form factor is itself larger, thereby increasing F_K. Numerically, the latter effect turns out to be more important. Interestingly, the twist-3 contribution which is entirely an $SU(3)_{\mathrm{fl}}$ violating effect is negligibly small, so that the m_s uncertainty is unimportant. Note that in the ratio of kaon and pion form factors some theoretical uncertainties (e.g., due to M^2 and scale dependence) cancel, leaving the major uncertainty in the Gegenbauer coefficient a_1. The unaccounted $SU(3)_{\mathrm{fl}}$ violating effects in higher twists can presumably be neglected within the present accuracy. If the kaon e.m. form factor is measured one would then be able to constrain/fit a_1.

Our final comment in this section concerns the neutral kaon e.m. form factor. It can be easily calculated from the same LCSR (42) if one replaces the u-quark by the d-quark in the initial correlation function, keeping in mind that the DAs of K^0 and K^+ are equal due to isospin symmetry. In particular, the leading twist-2 contribution is obtained replacing the u-quark charge 2/3 in (43) by the d-quark charge $-1/3$. As a result $F_{K^0}(Q^2)$ is a pure $SU(3)_{\mathrm{fl}}$ violating effect proportional to the integral over the difference $\varphi_K(u) - \varphi_K(\bar{u})$. The numerical result is small: $Q^2 F_{K^0}(Q^2) = 0.05$–$0.09\,\mathrm{GeV}^2$ (at $1 < Q^2 < 10\,\mathrm{GeV}^2$), implying that the measurement of this form factor is a difficult task.

We conclude that the LCSR method allows one to systematically account for $SU(3)_{\mathrm{fl}}$ breaking effects in the kaon form factors, and that these effects revealed by the ratio of K^+ and π^+ form factors are predicted to be quite noticeable.

6 The $K \to \pi$ form factor

As a final application of LCSR in this paper we consider the $K \to \pi$ form factor $f_{K\pi}^+$ defined by

$$\langle \pi^-(p-q) \mid \bar{s}\gamma_\mu u \mid K^0(p)\rangle$$
$$= 2 f_{K\pi}^+(q^2) p_\mu - (f_{K\pi}^+(q^2) - f_{K\pi}^-(q^2)) q_\mu. \qquad (47)$$

As explained in Sect. 2 this form factor can be calculated from the correlation function (1) in two different ways: either from the vacuum-to-pion or from the vacuum-to-kaon correlation functions. Both calculations are valid only at sufficiently large spacelike momentum transfer $Q^2 \geq 1\,\mathrm{GeV}^2$, whereas the form factor $f_{K\pi}^+$ is measurable only at timelike momenta where the LCSR method is not applicable, e.g., at $0 < q^2 < (m_K - m_\pi)^2$ in the K_{e3} decays, or at $(m_K + m_\pi)^2 < q^2 < m_\tau^2$ in $\tau \to K\pi\nu$ decays.

Nevertheless, one is able to use the fact that the LCSR obtained for two different settings yield one and the same physical parameter and derive useful constraints on the light-cone DAs of the pion and kaon involved in both sum rules. To explain this idea we explicitly write down the LCSR obtained from the vacuum-to-pion correlator:

$$f_{K\pi}^+(Q^2) = \frac{f_\pi}{f_K} \int_{u_0^K}^1 \mathrm{d}u \varphi_\pi(u,\mu)$$

[6] As noted above this is only consistent for the twist-3 part of the sum rule. The accuracy of the twist-4 part can be improved further if one determines the non-perturbative parameters entering the kaon twist-3 and -4 DAs from the corresponding two-point sum rules in the kaon channel taking into account $SU(3)_{\mathrm{fl}}$ violation, a task for future work

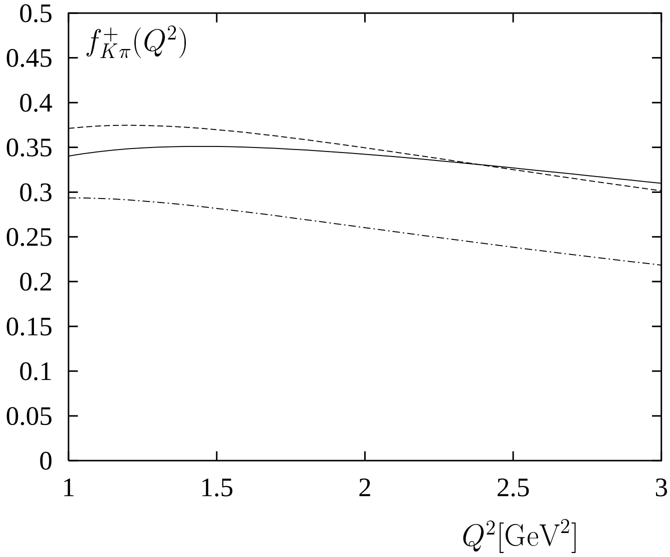

Fig. 6. The $K \to \pi$ form factor at spacelike region calculated with twist-2 accuracy: from (48) with the asymptotic pion DA (*solid*) and from (49) with the kaon DA including the $SU(3)_{\mathrm{fl}}$ violating correction $\sim a_1$ (*dashed*) and at $a_1 = 0$ (*dash-dotted*)

$$\times \exp\left(-\frac{\bar{u}Q^2}{uM^2} + \frac{m_K^2}{M^2}\right) + \cdots \qquad (48)$$

Using instead the vacuum-to-kaon correlator, one gets

$$f^+_{K\pi}(Q^2) = \frac{f_K}{f_\pi}\int_{u_0^\pi}^1 \mathrm{d}u \varphi_K(\bar{u},\mu)$$
$$\times \exp\left(-\frac{\bar{u}Q^2}{uM^2} - \frac{\bar{u}m_K^2}{M^2}\right) + \cdots, \qquad (49)$$

In the region $1 < Q^2 < 3\,\mathrm{GeV}^2$ and at $M^2 \sim 1\,\mathrm{GeV}^2$ both sum rules are valid and the higher twist and $O(\alpha_\mathrm{s})$ contributions denoted by ellipses are small, so that we may neglect them for the sake of simplicity. Equating (48) and (49) in this region one may constrain the non-asymptotic coefficients. In particular, the rate of $SU(3)_{\mathrm{fl}}$ breaking asymmetry in the kaon DA φ_K can be estimated if the pion DA φ_π is determined with sufficient accuracy.

As a numerical illustration, in Fig. 6 we compare the r.h.s. of (48) and (49) calculated respectively with the asymptotic pion DA and with the kaon DA adopted in our calculation of F_K in the previous section, that is, with $a_1(1\,\mathrm{GeV}) = -0.17$ and all $a_{n>1} = 0$. The resulting agreement of two different sum rules is non-trivial and gives confidence in the whole procedure and especially in the choice of duality thresholds in both pion and kaon channels. Note that the agreement is violated if we put $a_1 = 0$.

7 Conclusions

In this paper, we have studied the correlation functions of light-quark currents used to derive the LCSR for the pion and kaon form factors. We have demonstrated that the Ward identities for these correlators yield relations between DAs of twist 4, a new alternative to using the QCD equation of motions. On the phenomenological side, we have corrected the expression for the twist-4 contribution to the LCSR for the pion form factor. The form factor calculated with the purely asymptotic pion DA is generally consistent with the recent Jefferson Lab data. On the other hand, constraining the non-asymptotic part of the pion twist-2 DA in terms of separate Gegenbauer coefficients demands more data at intermediate momentum transfers, $1 < Q^2 < 10\,\mathrm{GeV}^2$ and largely depends on the particular ansatz adopted for this DA. A recent study of a similar problem for the $\gamma^*\gamma^* \to \pi^0$ form factor can be found in [22].

We have presented the first LCSR prediction for the kaon e.m. form factor and demonstrated that within the sum rule approach the $SU(3)_{\mathrm{fl}}$ violating difference between kaon and pion form factors is systematically calculable in powers of the strange quark mass. It has been shown that useful complementary information concerning the kaon DA can be obtained from a comparison of the two independent sum rules for the $K \to \pi$ form factor. In general, our results support the point of view that LCSR for the pion and kaon form factors combined with sufficiently precise data on these form factors represent a very useful tool for probing the pion and kaon light-cone distribution amplitudes.

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Appendix

A Light-cone expansion of the quark propagator

The expansion of the quark propagator with a non-zero mass m near the light-cone $(x_1 - x_2)^2 = 0$ reads (see e.g. [23]):

$$S(x_1,x_2,m) \equiv -\mathrm{i}\langle 0|T\{q(x_1)\bar{q}(x_2)\}|0\rangle$$
$$= \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k(x_1-x_2)}$$
$$\times \left\{ \frac{\not{k}+m}{k^2-m^2} - \int_0^1 \mathrm{d}v G^{\mu\nu}(vx_1 + (1-v)x_2) \right. \qquad (A.1)$$
$$\left. \times \left[\frac{1}{2}\frac{\not{k}+m}{(k^2-m^2)^2}\sigma_{\mu\nu} - \frac{1}{k^2-m^2}v(x_1-x_2)_\mu\gamma_\nu\right]\right\},$$

with $G^{\mu\nu} = g_s G^a_{\mu\nu}(\lambda^a/2)$, $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$. At $m = 0$, after the integration over k this expression reduces to the propagator derived in [24] (see also [7]):

$$S(x_1, x_2, 0) = \frac{\not x_1 - \not x_2}{2\pi^2[(x_1-x_2)^2]^2} - \frac{1}{16\pi^2(x_1-x_2)^2}\int_0^1 \mathrm{d}v G^{\mu\nu}(vx_1 + (1-v)x_2) \times [(\not x_1 - \not x_2)\sigma_{\mu\nu} - 4\mathrm{i}v(x_1-x_2)_\mu\gamma_\nu]. \quad \text{(A.2)}$$

B Light-cone distribution amplitudes

The light-cone DAs of the pseudoscalar meson $P = \pi, K$ are defined according to [10,11]. The matrix element of the axial-vector bilocal operator is expanded around the light-cone ($x_1^2 = x_2^2 = (x_1-x_2)^2 = 0$):

$$\langle 0|\bar q_2(x_2)\gamma_\mu\gamma_5 q_1(x_1)|P(p)\rangle = f_P \int_0^1 \mathrm{d}u \mathrm{e}^{-\mathrm{i}upx_1 - \mathrm{i}\bar u p x_2} \times \left\{ \mathrm{i}p_\mu \left(\varphi_P(u) + (x_1-x_2)^2 g_{1P}(u)\right) + \left((x_1-x_2)_\mu - \frac{p_\mu(x_1-x_2)^2}{p(x_1-x_2)}\right) g_{2P}(u)\right\}, \quad \text{(B.1)}$$

retaining the leading twist-2 DA $\varphi_P(u)$ and the twist-4 DAs $g_{1P}(u)$ and $g_{2P}(u)$, where u is the light-cone momentum fraction of the quark q_1. From the local limit of (B.1) one has the following normalization conditions:

$$\int_0^1 \mathrm{d}u\varphi_P(u) = 1, \quad \int_0^1 \mathrm{d}u g_{2P}(u) = 0. \quad \text{(B.2)}$$

The twist-3 quark–antiquark DAs φ_{pP} and $\varphi_{\sigma P}$ and the quark–antiquark–gluon DA φ_{3P} are defined as follows:

$$\langle 0|\bar q_2(x_2)\mathrm{i}\gamma_5 q_1(x_1)|P(p)\rangle = f_P\mu_P \int_0^1 \mathrm{d}u \mathrm{e}^{-\mathrm{i}upx_1 - \mathrm{i}\bar u p x_2}\varphi_{pP}(u),$$

$$\langle 0|\bar q_2(x_2)\sigma_{\alpha\beta}\gamma_5 q_1(x_1)|P(p)\rangle = \frac{\mathrm{i}f_P\mu_P}{6}\left(1 - \frac{m_P^2}{\mu_P^2}\right)[p_\alpha(x_1-x_2)_\beta - p_\beta(x_1-x_2)_\alpha] \times \int_0^1 \mathrm{d}u \mathrm{e}^{-\mathrm{i}upx_1 - \mathrm{i}\bar u p x_2}\varphi_{\sigma P}(u), \quad \text{(B.3)}$$

where $\mu_P = m_P^2/(m_{q_1} + m_{q_2})$, and

$$\langle 0|\bar q_2(x_2)\sigma_{\mu\nu}\gamma_5 G_{\alpha\beta}(vx_1 + \bar v x_2) q_1(x_1)|P(p)\rangle = \mathrm{i}f_{3P}\Big[(p_\alpha p_\mu g_{\beta\nu} - p_\beta p_\mu g_{\alpha\nu}) - (p_\alpha p_\nu g_{\beta\mu} - p_\beta p_\nu g_{\alpha\mu})\Big] \times \int \mathcal{D}\alpha_i \varphi_{3P}(\alpha_i,\mu) e^{-\mathrm{i}\alpha_1 p x_1 - \mathrm{i}\alpha_2 p x_2 - \mathrm{i}\alpha_3(vpx_1 + \bar v p x_2)}, \quad \text{(B.4)}$$

all these DAs being normalized to unity. Furthermore, the quark–antiquark–gluon twist-4 DAs are defined by the following matrix elements:

$$\langle 0|\bar q_2(x_2)\gamma_\mu\gamma_5 G_{\alpha\beta}(vx_1 + \bar v x_2) q_1(x_1)|P(p)\rangle = f_P \int \mathcal{D}\alpha_i e^{-\mathrm{i}\alpha_1 p x_1 - \mathrm{i}\alpha_2 p x_2 - \mathrm{i}\alpha_3(vpx_1 + \bar v p x_2)} \times \left\{ p_\mu \frac{p_\alpha(x_1-x_2)_\beta - p_\beta(x_1-x_2)_\alpha}{p(x_1-x_2)}\varphi_{\parallel P}(\alpha_i) + (g^\perp_{\mu\alpha}p_\beta - g^\perp_{\mu\beta}p_\alpha)\varphi_{\perp P}(\alpha_i)\right\}, \quad \text{(B.5)}$$

$$\langle 0|\bar q_2(x_2)\gamma_\mu \mathrm{i}\widetilde G_{\alpha\beta}(vx_1 + \bar v x_2) q_1(x_1)|P(p)\rangle = f_P \int \mathcal{D}\alpha_i \mathrm{e}^{-\mathrm{i}\alpha_1 p x_1 - \mathrm{i}\alpha_2 p x_2 - \mathrm{i}\alpha_3(vpx_1 + \bar v p x_2)} \times \left\{ p_\mu \frac{p_\alpha(x_1-x_2)_\beta - p_\beta(x_1-x_2)_\alpha}{p(x_1-x_2)}\widetilde\varphi_{\parallel P}(\alpha_i) + (g^\perp_{\mu\alpha}p_\beta - g^\perp_{\mu\beta}p_\alpha)\widetilde\varphi_{\perp P}(\alpha_i)\right\}, \quad \text{(B.6)}$$

where $\widetilde G_{\alpha\beta} = (1/2)\epsilon_{\alpha\beta\rho\lambda}G^{\rho\lambda}$ and the following abbreviations are used:

$$\mathcal{D}\alpha_i = \mathrm{d}\alpha_1 \mathrm{d}\alpha_2 \mathrm{d}\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$$

and

$$g^\perp_{\alpha\beta} = g_{\alpha\beta} - \frac{(x_1-x_2)_\alpha p_\beta + (x_1-x_2)_\beta p_\alpha}{p(x_1-x_2)}.$$

The distribution amplitudes are constructed [10] using the formalism of the conformal expansion. The most familiar example is the twist-2 DA [1]

$$\varphi_P(u,\mu) = 6u\bar u\left[1 + \sum_{n=1} a_n^P(\mu) C_n^{3/2}(u - \bar u)\right], \quad \text{(B.7)}$$

where the expansion goes in Gegenbauer polynomials $C_n^{3/2}$, the first four polynomials being

$$C_1^{3/2}(x) = 3x, \quad C_2^{3/2}(x) = -\frac{3}{2}(1 - 5x^2),$$
$$C_3^{3/2}(x) = -\frac{5}{2}x(3 - 7x),$$
$$C_4^{3/2}(x) = \frac{15}{8}(1 - 14x^2 + 21x^4). \quad \text{(B.8)}$$

The scale dependence is given in the leading order by

$$a_n^P(\mu_2) = [L(\mu_2,\mu_1)]^{-\gamma_n^{(0)}/\beta_0} a_n^P(\mu_1), \quad \text{(B.9)}$$

where $L(\mu_2,\mu_1)=\alpha_s(\mu_2)/\alpha_s(\mu_1)$, $\beta_0=11-(2/3)N_F$ and the anomalous dimensions are

$$\gamma_n^{(0)}=C_F\left[3+\frac{2}{(n+1)(n+2)}-4\left(\sum_{k=1}^{n+1}\frac{1}{k}\right)\right]. \quad (B.10)$$

For the pion, the coefficients a_n^π vanish at odd n in the isospin symmetry limit.

The twist-3 and -4 DAs have been derived in [10] using QCD equations of motion and a conformal expansion. In [11] the meson-mass corrections have been worked out. We present here the explicit expression for these DAs to next-to-leading accuracy in the conformal spin, including the meson-mass corrections (as explained in more detail in [10,11]) and using the original notations of [10]. Note that $SU(3)_{\rm fl}$ violating non-asymptotic corrections to these DAs (analogous to $a_1\neq 0$ for φ_K) are still missing and have to be worked out in the future.

The twist-3 DAs of the pseudoscalar meson, to next-to-leading order in conformal spin read

$$\varphi_{pP}(u)=1+\left(30\frac{f_{3P}}{\mu_P f_P}-\frac{5}{2}\frac{m_P^2}{\mu_P^2}\right)C_2^{1/2}(2u-1) \quad (B.11)$$
$$+\left(-3\frac{f_{3P}\omega_{3P}}{\mu_P f_P}-\frac{27}{20}\frac{m_P^2}{\mu_P^2}(1+6a_2^P)\right)C_4^{1/2}(2u-1),$$
$$\varphi_{\sigma P}(u)=6u\bar u\left\{1+\left(5\frac{f_{3P}}{\mu_P f_P}\left(1-\frac{1}{10}\omega_{3P}\right)-\frac{7}{20}\frac{m_P^2}{\mu_P^2}\left(1+\frac{12}{7}a_2^P\right)\right)C_2^{3/2}(2u-1)\right\}, \quad (B.12)$$
$$\varphi_{3P}(\alpha_i)=360\alpha_1\alpha_2\alpha_3^2\left(1+\frac{\omega_{3P}}{2}(7\alpha_3-3)\right). \quad (B.13)$$

The non-perturbative parameter f_{3P} is given by the matrix element which corresponds to the local limit in (B.4). The second parameter ω_{3P} determining the non-asymptotic parts of twist-3 DA is defined with the following matrix element (up to higher-twist corrections):

$$\langle 0|\bar q_2\sigma_{\mu\lambda}\gamma_5[D_\beta,G_{\alpha\lambda}]q_1-\frac{3}{7}\partial_\beta\bar q_2\sigma_{\mu\lambda}\gamma_5 G_{\alpha\lambda}q_1|P(p)\rangle$$
$$=\frac{3}{14}f_{3P}\omega_{3P}p_\alpha p_\beta p_\mu. \quad (B.14)$$

The scale dependence of the twist-3 parameters is given by

$$\mu_P(\mu_2)=[L(\mu_2,\mu_1)]^{-4/\beta_0}\,\mu_P(\mu_1),$$
$$f_{3P}(\mu_2)=[L(\mu_2,\mu_1)]^{(1/\beta_0)((7C_F/3)+N_c)}\,f_{3P}(\mu_1), \quad (B.15)$$
$$(f_{3P}\omega_{3P})(\mu_2)=[L(\mu_2,\mu_1)]^{(1/\beta_0)((7C_F/6)+(10N_c/3))}\times(f_{3P}\omega_{3P})(\mu_1). \quad (B.16)$$

Finally, the four twist-4 three-particle DAs defined in (B.5) and (B.6) are

$$\varphi_{\parallel P}(\alpha_i)=120\left(\delta_P^2\epsilon_P-\frac{9}{20}a_2^P m_P^2\right)(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3,$$
$$\varphi_{\perp P}(\alpha_i)=30(\alpha_1-\alpha_2)\alpha_3^2\times\left[\frac{\delta_P^2}{3}+2\delta_P^2\epsilon_P(1-2\alpha_3)+\frac{9}{40}a_2^P m_P^2(1-\alpha_3)\right],$$
$$\widetilde\varphi_{\parallel P}(\alpha_i)=-120\delta_P^2\alpha_1\alpha_2\alpha_3\left[\frac{1}{3}+\epsilon_P(1-3\alpha_3)\right],$$
$$\widetilde\varphi_{\perp}(\alpha_i)=30\alpha_3^2\left[\left(\frac{\delta_P^2}{3}+2\delta_P^2\epsilon_P(1-2\alpha_3)\right)(1-\alpha_3)+\frac{9}{40}a_2^P m_P^2(\alpha_1^2+\alpha_2^2-4\alpha_1\alpha_2)\right], \quad (B.17)$$

normalized as

$$\int\mathcal{D}\alpha_i\varphi_{\parallel P}(\alpha_i)=\int\mathcal{D}\alpha_i\varphi_{\perp P}(\alpha_i)=0,$$
$$-\int\mathcal{D}\alpha_i\widetilde\varphi_{\parallel P}(\alpha_i)=\int\mathcal{D}\alpha_i\widetilde\varphi_{\perp P}(\alpha_i)=\frac{\delta_P^2}{3}. \quad (B.18)$$

The corresponding two-particle DAs have the following expressions:

$$g_{1P}(u)=\frac{5}{2}\delta_P^2u^2\bar u^2+\left\{\frac{f_{3P}m_P^2}{4f_P\mu_P}[30(1-2u\bar u)-\omega_{3P}(3-u\bar u(27-56u\bar u))]+\frac{m_P^2}{320}\Big[5(25-29u\bar u)-12a_2^P(1-5u\bar u(19-52u\bar u))\Big]\right\}u\bar u$$
$$+\frac{1}{2}\left(\delta_P^2\epsilon-\frac{9}{20}a_2^P m_P^2\right)\Big[2u^3(10-15u+6u^2)\ln u+2\bar u^3(10-15\bar u+6\bar u^2)\ln\bar u+u\bar u(2+13u\bar u)\Big], \quad (B.19)$$
$$g_{2P}(u)=\left[\frac{10}{3}\delta_P^2+m_P^2\left(1+\frac{9}{8}a_2^P(1-7u\bar u)\right)-\frac{f_{3P}m_P^2}{f_P\mu_P}(10-\omega_{3P}(1-7u\bar u))\right]\bar u u(u-\bar u). \quad (B.20)$$

The non-perturbative parameter δ_P^2 is defined by

$$\langle 0|\bar q_2\widetilde G_{\alpha\mu}\gamma^\alpha q_1|P(p)\rangle=-\mathrm{i}\delta_P^2 f_P p_\mu, \quad (B.21)$$

with the scale dependence

$$\delta_P^2(\mu_2)=[L(\mu_2,\mu_1)]^{(8C_F)/(3\beta_0)}\,\delta_P^2(\mu_1), \quad (B.22)$$

whereas the second parameter ϵ_P determining the non-asymptotic corrections has the following definition in terms of a local matrix element (up to twist-5 corrections) [10,11]:

$$\langle 0|\bar q_2[D_\mu,\widetilde G_{\nu\xi}]\gamma^\xi q_1-\frac{4}{9}\partial_\mu\bar q_2\widetilde G_{\nu\xi}\gamma^\xi q_1|P(p)\rangle$$
$$=-\frac{8}{21}f_P\delta_P^2\epsilon_P\left(p_\mu p_\nu-\frac{1}{4}m_P^2 g_{\mu\nu}\right). \quad (B.23)$$

The corresponding scale dependence is

$$(\delta_P^2\epsilon_P)(\mu_2)=[L(\mu_2,\mu_1)]^{(10N_c)/(3\beta_0)}\,(\delta_P^2\epsilon_P)(\mu_1). \quad (B.24)$$

C Radiative corrections to the twist-2 pion form factor

Here, for completeness, we present the formula for the radiative correction to the twist-2 part of the sum rule for the pion form factor obtained in [7]:

$$F_\pi^{(2,\alpha_s)}(Q^2) = \int\limits_0^1 \mathrm{d}u\varphi_\pi(u,\mu)[\Theta(u-u_0)\mathcal{F}_{\rm soft}(u,M^2,s_0) + \Theta(u_0-u)\mathcal{F}_{\rm hard}(u,M^2,s_0)], \quad (C.1)$$

where

$$\begin{aligned}
&\mathcal{F}_{\rm soft}(u,M^2,s_0)\\
&= \frac{\alpha_s}{4\pi}C_F\left\{\exp\left(-\frac{\bar{u}Q^2}{uM^2}\right)\right.\\
&\times\left[-9+\frac{\pi^2}{3}+3\ln\frac{Q^2}{\mu^2}+3\ln\frac{\bar{u}Q^2}{u\mu^2}-\ln^2\frac{Q^2}{\mu^2}-\ln^2\frac{\bar{u}Q^2}{u\mu^2}\right]\\
&+\int\limits_{\bar{u}Q^2/u}^{s_0}\frac{\mathrm{d}sQ^2\mathrm{e}^{-s/M^2}}{u(Q^2+s)^3}\left[5s+Q^2\left(1+2\ln\frac{-\rho}{\mu^2}\right)\right.\\
&+2\left(\frac{Q^2}{\bar{u}}+s\right)\ln\frac{-\rho}{s}\\
&\left.+\frac{2Q^2}{u}\left(\frac{Q^2+s}{s}+\frac{2M^2+Q^2+s}{M^2}\ln\frac{-\rho}{s}\right)\ln\frac{-\rho}{\mu^2}\right]\\
&+\int\limits_0^{\bar{u}Q^2/u}\frac{\mathrm{d}sQ^2\mathrm{e}^{-s/M^2}}{u\bar{u}(Q^2+s)^3}\left[2u\left(Q^2-s+s\ln\frac{s}{\mu^2}\right)\right.\\
&+\left(-Q^2+5s+2(Q^2-s)\ln\frac{s}{\mu^2}\right.\\
&\left.\left.-\frac{s(Q^2+s)}{M^2}\left(-3+2\ln\frac{s}{\mu^2}\right)\right)\ln\frac{\rho}{\mu^2}\right]\\
&\left.+2\frac{u_0^2}{u^2}\mathrm{e}^{-s_0/M^2}\ln\frac{-\rho_0}{\mu^2}\ln\frac{u-u_0}{\bar{u}_0}\right\}, \quad (C.2)
\end{aligned}$$

and

$$\begin{aligned}
&\mathcal{F}_{\rm hard}(u,M^2,s_0)\\
&= \frac{\alpha_s}{4\pi}C_F\left\{\int\limits_0^{s_0}\frac{\mathrm{d}sQ^2\mathrm{e}^{-s/M^2}}{\bar{u}(Q^2+s)^3}\left[2\left(Q^2-s+s\ln\frac{s}{\mu^2}\right)\right.\right.\\
&+\frac{1}{u}\left(-Q^2+5s+2(Q^2-s)\ln\frac{s}{\mu^2}\right.\\
&\left.\left.-\frac{s(Q^2+s)}{M^2}\left(-3+2\ln\frac{s}{\mu^2}\right)\right)\ln\frac{\rho}{\mu^2}\right]\\
&\left.-\frac{u_0\bar{u}_0}{u\bar{u}}\mathrm{e}^{-s_0/M^2}\left(2\ln\frac{s_0}{\mu^2}-3\right)\ln\frac{\rho_0}{\mu^2}\right\}. \quad (C.3)
\end{aligned}$$

Here $\rho=\bar{u}Q^2-us$ and $\rho_0=\bar{u}Q^2-us_0=(1-u/u_0)Q^2$.

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